2017

(October)

PHYSICS

(Honours)

(Electrodynamics and Electronics—II)

[PHY-06 (T)]

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer Question No. **1** and *any* **four** from the rest

1. *(a)* The load resistance of an FET amplifier in the common source configuration is given to be 250 kΩ. Given the a.c. drain resistance of the device to be 100 kΩ and the transconductance to be 0·5 mAV^{-1}. Find the voltage gain and the output resistance of the amplifier. 3

   (b) A laser radiation beam of power 2000 watts is concentrated by a lens into cross-sectional area of about 10^{-10} m^2. Find the value of the Poynting vector. 3

Either

8. *(a)* Explain the following statements in FORTRAN : 2×3=6

(i) DIMENSION statement

(ii) FORMAT statement

(iii) END statement

(b) Develop the relevant flowchart and an algorithm for finding out the real, equal and imaginary roots of the equation $ax^2 + bx + c = 0$, where a, b and c are constants. 2+3=5

Or

(a) Illustrate with examples 'executable' and 'non-executable' statements. 2+2=4

(b) What are 'library functions' in FORTRAN? Illustrate with examples. 1+2=3

(c) Explain with illustrative examples the usage of the following FORTRAN statements : 2+2=4

(i) Input statement—formatted and unformatted

(ii) GO TO statement—unconditional GO TO statement and computed GO TO statement

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2019

(October)

PHYSICS

(Honours)

[PHY-06(T)]

(Electrodynamics, Electronics—II)

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer Question No. **1**, which is compulsory and *any* **four** from the rest

1. *(a)* Dielectric constant of a gas at NTP is 1.000074. Calculate *(i)* the induced dipole moment per unit volume and *(ii)* dipole moment per atom of the gas, when it is held in an external electric field of 3×10^4 V/m. 2+2=4

(b) A differential amplifier has an open circuit voltage gain of 100. The amplifier has a common input signal of 3·2 V to both terminals. This results in an output signal of 26 mV. Determine *(i)* the common-mode voltage gain and *(ii)* the CMRR in dB. 2+2=4

(c) Evaluate the following arithmetic FORTRAN statement for $I = 5$, $J = 7$ and $K = 10$

$$X = I * I / J - K * 2 / I$$ 2

(d) Simplify the following Boolean expression : 2

$$Y = 1 + A(B \cdot \bar{C} + BC + \overline{BC}) + A\bar{B}C + AC$$

2. (a) Write down Maxwell's equations in free space. Identify the symbols and give their empirical basis. 3

(b) Define the terms dielectric constant k and electric susceptibility χ_e. Prove the relation $k = 1 + \chi_e$. 1+1+2=4

(c) Derive Clausius-Mossotti equation. 4

3. (a) Derive an expression for Gauss's law in a dielectric medium. 3

(b) What are magnetic scalar potention ϕ and magnetic vector potential $\vec{A}$? If at any position the magnetic vector potential is $\vec{A} = 5(x^2 + y^2 + z^2)\hat{i}$, evaluate the magnetic field at that position. 2+2=4

(c) If $\vec{A}$ represents the magnetic vector potential and $\vec{J}$ the current density vector, then show that $\nabla^2 \vec{A} = -\mu_0 \vec{J}$ and identify the equation. 4

4. (a) Discuss the theories of reflection and transmission of a plane e.m. wave at a boundary of two dielectrics. 6

(b) Considering normal incidence of electromagnetic waves at the boundary between two dielectric media, derive the expressions for reflectance and transmittance. 5

5. (a) State two points of similarities and dissimilarities each between JFET and BJT. Explain the terms pinch-off voltage and shorted-gain drain current in a JFET circuit. 2+2+2+2=8

(b) What is a MOSFET? Mention its types. 2+1=3

6. (a) What is an operational amplifier (OP-AMP)? Draw the basic circuit of a differential amplifier and discuss its operation. 2+2+2=6

(b) What are gauge transformations? Discuss the significance of Lorentz gauge. 2+3=5

7. (a) With the help of a neat circuit diagram, explain the working of a Colpitts' oscillator. Write the expression for the frequency of oscillations for it. 2+2+1=5

(b) Using two's complement scheme, perform the following binary subtraction 1

1110011 − 1001111

(c) Convert $(43 \cdot 812)_{10}$ to binary. 1

(d) What is a digital comparator? Draw a 1-bit digital comparator. 2

(e) Explain the working of a multiplexer with the help of a logic circuit of 'two line to one line' multiplexer. 2

8. *(a)* Draw a flowchart for solving a quadratic equation. Develop an algorithm for the same and hence write a program in FORTRAN to solve a quadratic equation. 2+2+2=6

(b) Explain the usage of the following input-output statements in FORTRAN

(i) GO TO

(ii) IF THEN, ELSE, ENDIF 1½+1½=3

(c) What are non-executable statements in FORTRAN programming? Explain any one with example. 1+1=2

9. *(a)* Explain with examples *(i)* double-precision variable and *(ii)* logical constant. 1½+1½=3

(b) What are executable statements in FORTRAN programming? List a few and explain any one of them. 1½+1½=3

(c) Explain with illustration formatted and unformatted output statements in FORTRAN. 2+2=4

(d) Explain any one of the following FORTRAN statements : 1

(i) COMMON

(ii) DIMENSION

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(October)

PHYSICS

(Honours)

(Mathematical Physics, Quantum Mechanics)

[PHY-05 (T)]

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer Question No. **1** which is compulsory and *any* **four** from the rest

1. *(a)* Prove that $f(z) = z^3$ is analytic in entire z-plane. 2

(b) Define covariant and contravariant tensors. Using the property of Γ-function, evaluate $\int_0^\infty x^4 e^{-x^2}\, dx$. 1+1+2=4

(c) Evaluate the value of commutator $[p_x, x^n]$, where x is the position coordinate and p_x is the component of linear momentum along x-axis. 3

(d) If $\vec{A}$ and $\vec{B}$ are two vector operators which commute with Pauli's spin matrix $\vec{\sigma}$, prove that

$$(\vec{\sigma}\cdot\vec{A})(\vec{\sigma}\cdot\vec{B}) = \vec{A}\cdot\vec{B} + i\vec{\sigma}\cdot(\vec{A}\times\vec{B})$$ 3

2. *(a)* State and prove Gauss' divergence theorem of a vector function. 1+4=5

(b) Show that a square matrix can be expressed as a sum of two matrices, one symmetric and the other anti-symmetric. 3

(c) Find the eigenvalues and corresponding eigenvectors for the matrix

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$ 3

3. *(a)* What is an analytic function? Find the necessary condition for a function $f(z)$ to be analytic. 1+4=5

(b) State and prove Cauchy's integral formula. 1+3=4

(c) Use Cauchy's integral formula to evaluate

$$\int_C \frac{e^z}{(z-1)}\,dz$$

where C is the circle $|z| = 2$. 2

4. *(a)* For Legendre polynomials, prove that $\int_{-1}^{1} P_m(x)\,P_n(x)\,dx = 0$ if $m \neq n$. 4

(b) For Legendre polynomials, establish the recurrence formula

$$n\,P_n = (2n-1)x\,P_{n-1} - (n-1)\,P_{n-2}$$ 4

(c) Using the method of separation of variables, find the solution of

$$\frac{\partial u}{\partial x} + u = \frac{\partial u}{\partial t}$$

if $u = 4e^{-3x}$ at $t = 0$. 3

5. *(a)* Establish the continuity equation for matter waves. What is the physical significance of this equation? Also show that the probability current density is a real quantity. 3+1+2=6

(b) What are commutating and anti-commutating operators? Show that position coordinate x and component of linear momentum p_x are not commutating. 2+3=5

6. *(a)* What do you mean by eigenvalue, eigenvector (eigenfunction) and eigen-equation of an operator? Prove that eigenvalues of a Hermitian operator are real. 3+3=6

(b) Use operator method to establish the Heisenberg's uncertainty principle relation. 5

7. (a) Find the values of $[L_X, L_Y]$ and $[L_X, L^2]$. Is it possible to measure precisely the values of L^2 and three components of angular momentum $\vec{L}$ simultaneously? Justify your answer. 3+2+2=7

(b) State the postulates of quantum mechanics. 4

8. (a) $\vec{S}$ is the spin angular momentum and is related to Pauli's spin matrix $\vec{\sigma}$ as $\vec{S} = \frac{1}{2}\hbar\vec{\sigma}$. Prove that—

(i) $[\sigma_x, \sigma_y] = 2i\sigma_z$

(ii) $[\sigma_x, \sigma_y]_+ = 0$

(iii) $\sigma_x\sigma_y = i\sigma_z$ 2+1+1=4

(b) Write Schrödinger equation for the hydrogen atom in spherical polar coordinates. Separate the radial and angular parts and solve the radial equation to show that energy of the hydrogen atom is quantized. 1+1+5=7

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2018

(October)

PHYSICS

(Honours)

(Mathematical Physics, Quantum Mechanics)

[PHY–05 (T)]

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer Question No. **1** which is compulsory and *any* **four** from the rest

1. *(a)* Use Gauss' divergence theorem to prove that $\iint_S (\nabla\phi \times \nabla\psi) \cdot \hat{n}\, dS = 0$. 3

(b) Evaluate *(i)* $\Gamma(-\frac{1}{2})$ and *(ii)* $\Gamma(1)$. 3

(c) Find out the expectation value of the linear momentum of a particle moving in a one-dimensional box of width a. The normalized wave function of the particle is $\psi(x) = \sqrt{\frac{2}{a}} \sin \frac{n\pi}{a} x$. 3

(d) A and B are two Hermitian operators. Prove that AB will also be Hermitian if A and B commute. 3

2. (a) Define a Hermitian matrix. Prove that diagonal elements of a Hermitian matrix are real. 1+2=3

(b) Find the eigenvalues and corresponding eigenvectors for the matrix

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$ 4

(c) Use Stokes' theorem to evaluate $\oint_C \vec{F} \cdot d\vec{R}$ where $\vec{F} = y\hat{i} + xz^3\hat{j} - zy^3\hat{k}$, C is the circle $x^2 + y^2 = 4$ and $z = 1 \cdot 5$. 4

3. (a) Prove that Kronecker delta (δ_{ij}) is a mixed tensor of rank two. 3

(b) Write the transformation equation of a contravariant tensor. If $\dot{x}$ and $\dot{y}$ are the components of velocity in Cartesian coordinate system, find the components of velocity in polar coordinate system. 5

(c) Prove that $\Gamma(n+1) = n\Gamma(n)$. 3

4. (a) State and prove Cauchy's theorem. 1+3=4

(b) Use Cauchy's integral formula to evaluate

$$\oint_C \frac{\cos \pi z}{z-1} dz$$

where C is the circle $|z| = 3$. 3

(c) Use Cauchy's residue theorem to evaluate

$$\int_C \frac{1-2z}{z(z-1)(z-2)} dz$$

where C is the circle $|z| = 1 \cdot 5 \left(\frac{3}{2}\right)$. 4

5. Generating function of Legendre polynomial is given by $(1 - 2xt + t^2)^{-\frac{1}{2}}$. Use this function to—

(a) find the expression for $P_2(x)$;

(b) prove that $P_n(1) = 1$;

(c) show that $\int_{-1}^{1} [P_n(x)]^2 dx = \frac{2}{2n+1}$. 4+3+4=11

6. (a) State and prove Ehrenfest's theorem. 8

(b) Prove that linear momentum operator is a Hermitian operator. 3

7. (a) A potential step is given by

$$V(x) = 0 \quad x < 0$$
$$= V_0 \quad x \geq 0$$

For a particle of energy $E (< V_0)$ incident from left on the potential step the solutions of Schrödinger equation are given by

$$\psi(x) = A e^{i\alpha x} + B e^{-i\alpha x} \quad \text{for} \quad x < 0$$
$$= C e^{-\beta x} \quad \text{for} \quad x > 0$$

For the system given above, calculate the reflection and transmission coefficient. 2½×2=5

(b) In the region $x > 0$, calculate $\langle x \rangle$, $\langle x^2 \rangle$ and hence prove that $\Delta x \sim \frac{1}{\beta}$. 2+2+2=6

8. (a) Write the eigenvalue equation for L^2 in spherical polar coordinate system. Solve this equation to find an expression for the eigenvalue of L^2 for $L_z = 0$. 1+6=7

(b) Prove that while L^2 and L_z are simultaneously measurable with absolute accuracy, L_z and L_x are not measurable with same accuracy level. 2+2=4

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2019

(October)

PHYSICS

(Honours)

[PHY-05(T)]

(Mathematical Physics, Quantum Mechanics)

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer Question No. **1** which is compulsory and *any* **four** from the rest

1. *(a)* Define curl of a vector field $\vec{A}$. When is $\vec{A}$ irrotational? 1+1=2

(b) Show that every eigenvalue of a Hermitian operator is real. 4

(c) Solve $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ by the method of seperation of variables if

$$u(0, y) = 8e^{-3y}$$ 4

(d) Define covariant and contravariant tensors. 2

2. (a) State and prove Gauss divergence theorem of a vector function. 1+4=5

(b) Show that

$$\int_S \vec{B}\cdot\hat{n}\, dS = 0$$

if $\vec{B} = \vec{\nabla}\times\vec{A}$ for any closed surface S and $\hat{n}$ being the unit normal outward vector to S. 3

(c) Find the eigenvalues of the matrix

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$ 3

3. (a) If $f(z) = u(x,y) + iv(x,y)$ is an analytic function in a domain, then obtain the Cauchy-Riemann conditions

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$ 4

(b) State and prove Cauchy's integral formula. Use this formula to evaluate

$$\int_C \frac{z\,dz}{(9-z^2)(z+1)}$$

where C is a circle $|z| = 2$. 3+4=7

4. (a) In Legendre's polynomial, use the generating function

$$\left(1-2xz+z^2\right)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} z^n P_n(x)$$

to obtain the recurrence relation

$$nP_n = (2n-1)\,n\,P_{n-1} - (n-1)P_{n-2}$$ 4

(b) Obtain the condition of orthoganality of Legendre's polynominal. 4

(c) Write down Rodregue's formula for Legendre's polynomial. Use this formula to show that

$$P_2(x) = \frac{1}{2}\left(3x^2-1\right)$$ 1+2=3

5. (a) Define gamma function and hence show that

(i) $\Gamma n+1 = n\Gamma n$; (ii) $\Gamma\frac{1}{2} = \sqrt{\pi}$ 1+2+2=5

(b) What is beta function? Obtain the relation between gamma function and beta function. 1+5=6

6. (a) Derive Heisenberg's uncertainity relation

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$

by using operator method. 7

(b) What is a Hermitian operator? Show that the product of two Hermitian operators is Hermitian only if they commute. 1+3=4

7. *(a)* Determine the energy level and the corresponding normalised eigen-functions of a particle in one-dimensional potential well of the form

$$V(x) = \infty \text{ for } x < 0 \text{ and for } x > a$$
$$= 0 \text{ for } 0 < x < a$$

What are the boundary conditions for the problem? Is the wave function continuous everywhere? 3+3+1+1=8

(b) Show that every tensor of second rank can be resolved into symmetric and anti-symmetric tensors. 3

8. *(a)* Find the values of $[L_x, P_x]$ and $[L^2, L_y]$. 1½+1½=3

(b) If σ is the Pauli's spin matrix, show that

$$\left[\sigma_x, \sigma_y\right] = 2i\sigma_z$$ 2

(c) Write the Schrödinger equation for hydrogen atom in spherical polar coordinates. Solve the radial part of this equation to obtain the eigenvalues of energy. 1+5=6

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